Economics 257

November 17, 2025 **R. Davidson**

Assignment 3

1. This exercise is closely related to one on the previous assignment. Let p denote the proportion of individuals in a population that suffer from or carry a disease. Let p_+ be the probability of a diagnostic test returning a positive result with an individual who does carry the disease, and p_- the probability of a positive result with an individual who does not. What is the probability that an individual for whom the test gives a positive result does indeed carry the disease?

2. Suppose that Z_1 and Z_2 are two independent standard Normal random variables. Define two more variables, X_1 and X_2, as deterministic functions of Z_1 and Z_2 as follows:

$$X_1 = \sigma_1 Z_1, \qquad X_2 = \sigma_2(\rho Z_1 + \sqrt{1-\rho^2}\,Z_2).$$

Determine the variances of X_1 and X_2, their covariance, and their correlation.

What is the expectation of X_2 conditional on X_1? (Recall that this conditional expectation is a deterministic function of X_1.) Show that the variance of X_2 conditional on X_1 is $\sigma_2^2(1-\rho^2)$. Explain why this means that knowledge of the realisation of X_1 leads to more precise information about the realisation of X_2.

What is the marginal density of the distribution of X_2 conditional on X_1?

3. Let U be a random variable that has the U(0,1) distribution, and define X to be $-\log U$. Show that the both the expectation and the variance of X are equal to 1. In case you have not seen how to integrate the logarithmic function, note that:

$$\frac{\mathrm{d}}{\mathrm{d}x}(x\,\log x - x) = \log x \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}x}\big[x(\log x)^2 - 2x\log x + 2x\big] = (\log x)^2.$$

Show that the probability that X is greater than some number ℓ is $\mathrm{e}^{-\ell}$. For $\ell = 4$, how does this exact probability compare with the upper bounds given by the two Theorems at the end of Chapter 7, one of which is Chebychev's inequality?

4. Consider the binomial distribution with parameters n and p and let the random variable X have this distribution. It has the following probability mass function:

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k} = \frac{n\,!}{k\,!(n-k)!} p^k (1-p)^{n-k}, \qquad k = 0,1,2,\ldots,n.$$

If B_i, $i = 1,2,\ldots,n$ are independent Bernoulli random variables, with $P(B_i = 1) = p$, then we showed that X can be constructed as

$$X = \sum_{i=1}^{n} B_i.$$

Use this result to obtain the expectation and variance of X.