Economics 257

November 24, 2025   **R. Davidson**

Assignment 4

1.   A random variable X has the density function:

$$f_X(x) = \begin{cases} 0 & \text{for } x < 3; \\ x - 3 & \text{for } 3 \le x \le 4; \\ 5 - x & \text{for } 4 < x < 5; \\ 0 & \text{for } x > 5. \end{cases}$$

Using the expression for the expectation (mean) of a continuous random variable:

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} x\, f_X(x)\, \mathrm{d}x,$$

and treating the intervals $3 \le x \le 4$ and $4 < x \le 5$ separately, find the expectation and variance of X.

2.   For any random variable X, discrete or continuous, for which the first two moments exist, and for fixed real numbers a and b, show that

$$\mathrm{Var}(a + bX) = b^2\, \mathrm{Var}(X).$$

3.   For a random variable X which has finite fourth moment, expectation μ and variance σ^2, let $\kappa(X) \equiv \mathrm{E}\big((X - \mu)^4\big)/\sigma^4$ be a measure of the kurtosis of X. For a and b non-random real numbers, show that

$$\kappa(a + bX) = \kappa(X).$$

Explain your answer.

4.   Let X_1 and X_2 be a pair of random variables. Show that the covariance between the random variables $X_1 + X_2$ and $X_1 - X_2$ is zero if and only if X_1 and X_2 have the same variance.

5.   Scores on an achievement test are known to be normally distributed, with expectation 420 and standard deviation 80.

(a) For a randomly chosen person taking this test, what is the probability of a score between 400 and 480?

(b) What is the minimum score needed in order to be in the top 10% of all people taking the test?

(c) For a randomly chosen individual, state, without doing the calculations, in which of the following ranges his or her score is most likely to be: 400–440, 440–480, 480–520 or 520–560.

(d) In which of the ranges listed in (c) is the individual's score least likely to be?

(e) Two people taking the test are chosen at random. What is the probability that at least one of them scores more than 500 points?