Economics 662

September 19, 2024 **R. Davidson**

Assignment 1

This is Exercise 2.15 from *Foundations of Econometrics*.

1. Consider the linear regression models

$$H_1: \quad y_t = \beta_1 + \beta_2 x_t + u_t \quad \text{and}$$
$$H_2: \quad \log y_t = \gamma_1 + \gamma_2 \log x_t + u_t.$$

Suppose that the data are actually generated by H_2, with $\gamma_1 = 1.5$ and $\gamma_2 = 0.5$, and that the value of x_t varies from 10 to 110 with an average value of 60. Ignore the disturbances and consider the deterministic relations between y_t and x_t implied by the two models. Find the values of β_1 and β_2 that make the relation given by H_1 have the same level and the same value of $\mathrm{d}y_t/\mathrm{d}x_t$ as the level and value of $\mathrm{d}y_t/\mathrm{d}x_t$ implied by the relation given by H_2 when it is evaluated at the average value of the regressor.

Using the deterministic relations, plot y_t as a function of x_t for both models for the range $10 \le x_t \le 110$. Also plot $\log y_t$ as a function of $\log x_t$ for both models for the same range of x_t. How well do the two models approximate each other in each of the plots?

2. A vector in E^n can be **normalized** by multiplying it by the reciprocal of its norm. Show that, for any $\boldsymbol{x} \in E^n$ with $\boldsymbol{x} \neq \boldsymbol{0}$, the norm of $\boldsymbol{x}/\|\boldsymbol{x}\|$ is 1.

Now consider two vectors $\boldsymbol{x}, \boldsymbol{y} \in E^n$. Compute the norm of the sum and of the difference of $\boldsymbol{x}$ normalized and $\boldsymbol{y}$ normalized, that is, of

$$\frac{\boldsymbol{x}}{\|\boldsymbol{x}\|} + \frac{\boldsymbol{y}}{\|\boldsymbol{y}\|} \quad \text{and} \quad \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|} - \frac{\boldsymbol{y}}{\|\boldsymbol{y}\|}.$$

By using the fact that the norm of any nonzero vector is positive, prove the Cauchy-Schwartz inequality:

$$|\boldsymbol{x}^\top \boldsymbol{y}| \le \|\boldsymbol{x}\|\,\|\boldsymbol{y}\|.$$

Show that this inequality becomes an equality when $\boldsymbol{x}$ and $\boldsymbol{y}$ are parallel.

3. Consider the autoregressive model

$$y_t = \beta_1 + \beta_2 y_{t-1} + u_t, \qquad t = 2, \ldots, n.$$

Consider a DGP for which $\beta_1 = 2$, $\beta_2 = 0.5$, and $u_t \sim \mathrm{NID}(0, 1)$, and $y_0 = 1.5$.. Carry out a simulation experiment in order to estimate the bias of the OLS estimator of β_2, and that of the OLS estimator of β_1 for this DGP, with sample sizes $n = 20, 50, 100, 200, 500$. Do your simulation results suggest that either of the OLS estimators is (i) unbiased, or (ii) consistent? If the estimators appear biased, use your simulation results to estimate the bias. How do these estimates vary with the sample size?