# Economics 662

**September 23, 2025** **R. Davidson**

## Assignment 1

**1.** A vector in $E^n$ can be **normalized** by multiplying it by the reciprocal of its norm. Show that, for any $\boldsymbol{x} \in E^n$ with $\boldsymbol{x} \neq \boldsymbol{0}$, the norm of $\boldsymbol{x}/\|\boldsymbol{x}\|$ is 1.

Now consider two vectors $\boldsymbol{x}, \boldsymbol{y} \in E^n$. Compute the norm of the sum and of the difference of $\boldsymbol{x}$ normalized and $\boldsymbol{y}$ normalized, that is, of

$$\frac{\boldsymbol{x}}{\|\boldsymbol{x}\|} + \frac{\boldsymbol{y}}{\|\boldsymbol{y}\|} \quad \text{and} \quad \frac{\boldsymbol{x}}{\|\boldsymbol{x}\|} - \frac{\boldsymbol{y}}{\|\boldsymbol{y}\|}.$$

By using the fact that the norm of any nonzero vector is positive, prove the Cauchy-Schwartz inequality:

$$|\boldsymbol{x}^\top \boldsymbol{y}| \leq \|\boldsymbol{x}\| \, \|\boldsymbol{y}\|.$$

Show that this inequality becomes an equality when $\boldsymbol{x}$ and $\boldsymbol{y}$ are parallel.

**2.** Consider the autoregressive model

$$y_t = \beta_1 + \beta_2 y_{t-1} + u_t, \qquad t = 2, \ldots, n.$$

Consider a DGP for which $\beta_1 = 2$, $\beta_2 = 0.5$, and $u_t \sim \text{NID}(0, 1)$, and $y_0 = 1.5$.. Carry out a simulation experiment in order to estimate the bias of the OLS estimator of $\beta_2$, and that of the OLS estimator of $\beta_1$ for this DGP, with sample sizes $n = 20, 50, 100, 200, 500$. Do your simulation results suggest that either of the OLS estimators is (i) unbiased, or (ii) consistent? If the estimators appear biased, use your simulation results to estimate the bias. How do these estimates vary with the sample size?