Economics 662

October 8, 2024 **R. Davidson**

Assignment 2

1. An exercise about oblique projections. Let $\boldsymbol{X}$ and $\boldsymbol{W}$ be two $n \times k$ matrices, with $n > k$, and such that $\boldsymbol{W}^\top\boldsymbol{X}$ is non-singular. These two matrices define the oblique projection matrix

$$\boldsymbol{P} = \boldsymbol{X}(\boldsymbol{W}^\top\boldsymbol{X})^{-1}\boldsymbol{W}^\top.$$

The matrix $\boldsymbol{P}$ is clearly idempotent, but not symmetric in general.

Characterise the image of the $n \times n$ matrix $\boldsymbol{P}$. What is the dimension of this image? Characterise the nullspace of $\boldsymbol{P}$ – that is, what is the subspace the elements of which are mapped to zero by the action of $\boldsymbol{P}$? What is the dimension of this subspace?

Provide a necessary and sufficient condition on the matrix $\boldsymbol{W}$ for the action of $\boldsymbol{P}$ to be equivalent to that of the orthogonal projection matrix $\boldsymbol{P}_{\boldsymbol{X}}$.

2. The class of estimators considered by the Gauss-Markov Theorem can be written as $\hat{\boldsymbol{\beta}} = \boldsymbol{A}\boldsymbol{y}$, with $\boldsymbol{A}\boldsymbol{X} = \mathbf{I}$. Show that this class of estimators is in fact identical to the class of estimators of the form

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{W}^\top\boldsymbol{X})^{-1}\boldsymbol{W}^\top\boldsymbol{y},$$

where $\boldsymbol{W}$ is a matrix of exogenous variables such that $\boldsymbol{W}^\top\boldsymbol{X}$ is nonsingular.

3. Generate a figure like Figure 3.15 for yourself. Begin by drawing 100 observations of a regressor x_t from the $\mathrm{N}(0, 1)$ distribution. Then compute and save the h_t for a regression of any regressand on a constant and x_t. Plot the points (x_t, h_t), and you should obtain a graph similar to the one in Figure 3.15.

Now add one more observation, x_{101}. Start with $x_{101} = \bar{x}$, the average value of the x_t, and then increase x_{101} progressively until $x_{101} = \bar{x} + 20$. For each value of x_{101}, compute the leverage measure h_{101}. How does h_{101} change as x_{101} gets larger? Why is this in accord with the result that $h_t = 1$ if the regressors include the dummy variable $\boldsymbol{e}_t$?