Economics 662

November 19, 2024 **R. Davidson**

Assignment 4

This assignment is based on Exercises 8.10 and 8.11 of the textbook.

Use the DGP

$$\boldsymbol{y} = \boldsymbol{x}\beta_0 + \sigma_u \boldsymbol{u},$$

$$\boldsymbol{x} = \boldsymbol{w}\pi_0 + \sigma_v \boldsymbol{v},$$

to generate at least 1000 sets of simulated data for $\boldsymbol{x}$ and $\boldsymbol{y}$ with sample size $n = 10$, using normally distributed disturbances and parameter values $\sigma_u = \sigma_v = 1$, $\pi_0 = 1$, $\beta_0 = 0$, and the correlation of u_t and v_t, $t = 1, \dots, n$, $\rho = 0.5$. For the exogenous instrument $\boldsymbol{w}$, use independent drawings from the standard normal distribution, and then rescale $\boldsymbol{w}$ so that $\boldsymbol{w}^\top \boldsymbol{w}$ is equal to n, rather than 1 as in Section 10.4.

For each simulated data set, compute the IV estimator:

$$\hat{\beta}_{\text{IV}} = (\boldsymbol{w}^\top \boldsymbol{x})^{-1} \boldsymbol{w}^\top \boldsymbol{y} = \beta_0 + \sigma_u (\boldsymbol{w}^\top \boldsymbol{x})^{-1} \boldsymbol{w}^\top \boldsymbol{u}.$$

Then draw the empirical distribution of the realisations of the estimator on the same plot as the CDF of the normal distribution with expectation zero and variance $\sigma_u^2 / n\pi_0^2$. Explain why this is an appropriate way to compare the finite-sample and asymptotic distributions of the estimator.

In addition, for each simulated data set, compute the OLS estimator, and plot the EDF of the realisations of this estimator on the same axes as the EDF of the realisations of the IV estimator.

Redo the above exercise for a sample size of $n = 100$. If you have enough computer time available, redo it yet again for $n = 1000$, in order to see how quickly or slowly the finite-sample distribution tends to the asymptotic distribution.