# Economics 662

**December 5, 2025** **R. Davidson**

## Final Examination

*Please send your completed exam to Miroslav Zhao <***miroslav.zhao@mail.mcgill.ca***>, or upload it to myCourses, by 08.00 on Saturday December 6. As for the assignments, please submit two files per student: one, which should be a PDF file, with your written answers, and another, which may or may not be a simple text file, with your computer code. These files must be all your own work. You may make use of whatever non-human resources you wish, provided that you acknowledge them with a suitable citation, but you must not ask for or receive any help from any other person. Note that the questions all have different weights in the marking scheme.*

*All students in this course have the right to submit in English or in French any written work that is to be graded.*

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*Answer all seven questions in this exam. They do not all carry equal weight.*

*Faites tous les sept exercices de cet examen. Ils ont des poids différents.*

**1.** Consider estimation by instrumental variables of the linear regression model

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{u} \tag{1}$$

with an $n \times k$ matrix $\boldsymbol{X}$ of explanatory variables and an $n \times l$ matrix $\boldsymbol{W}$ of instrumental variables with $l > k$. Let $\hat{\boldsymbol{u}}$ be the vector of residuals from this estimation procedure. Show that $nR^2$ from the following artificial regression

$$\hat{\boldsymbol{u}} = \boldsymbol{Wb} + \text{residuals}$$

is equal to the Sargan test statistic, that is, the minimized IV criterion function for the model (1) divided by the IV estimate of the error variance for that model.

Consider the following OLS regression, where the matrix $\boldsymbol{X}$ of explanatory variables is partitioned as $\boldsymbol{Z} = [\boldsymbol{Z} \quad \boldsymbol{Y}]$, with $\boldsymbol{Z}$ the exogenous variables that are in both $\boldsymbol{X}$ and $\boldsymbol{W}$, and $\boldsymbol{Y}$ the potentially endogenous explanatory variables:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{M_W Y\zeta} + \boldsymbol{u}.$$

Show that an $F$ test of the restrictions $\boldsymbol{\zeta} = \mathbf{0}$ in this regression is numerically identical to the $F$ test for $\boldsymbol{\delta} = \mathbf{0}$ in the regression

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{P_W Y\delta} + \boldsymbol{u} \tag{2}$$

that is conventionally used in order to implement the Durbin-Wu-Hausman test. Show further that the OLS estimator of $\boldsymbol{\beta}$ from (2) is identical to the estimator $\hat{\boldsymbol{\beta}}_{\text{IV}}$ obtained by estimating (1) by instrumental variables.

**2.** Generate $N$ simulated data sets, where $N$ is between 1000 and $1,000,000$, depending on the capacity of your computer, from each of the following two data generating processes:

$$\text{DGP 1:} \quad y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + u_t, \quad u_t \sim \text{N}(0,1)$$
$$\text{DGP 2:} \quad y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + u_t, \quad u_t \sim \text{N}(0,\sigma_t^2), \;\; \sigma_t^2 = \big(\text{E}(y_t)\big)^2.$$

There are 50 observations, $\boldsymbol{\beta} = [1 \vdots 1 \vdots 1]$, and the data on the exogenous variables are to be found in the file **https://russell-davidson.research.mcgill.ca/data/mw.data**. These data were used in the MacKinnon and White (1985) paper in which $\text{HC}_3$ was developed.

For each of the two DGPs and each of the $N$ simulated data sets, construct .95 confidence intervals for $\beta_1$ and $\beta_2$ using the usual OLS covariance matrix and the HCCMEs $\text{HC}_0$, $\text{HC}_1$, $\text{HC}_2$, and $\text{HC}_3$. The OLS interval should be based on Student's $t$ distribution with 47 degrees of freedom, and the others should be based on the $\text{N}(0,1)$ distribution. Report the proportion of the time that each of these confidence intervals included the true values of the parameters.

**3.** This question and the next deal with a panel-data model. See section 9.10 of the textbook for more details.

The file **https://russell-davidson.research.mcgill.ca/data/e662.dec25.data**

contains data on three variables, arranged as three $m \times T$ matrices, with $m = 10$, $T = 12$. The first is the dependent variable $\boldsymbol{y}$, and there follow $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, two explanatory variables. Consider the panel data model:

$$y_{it} = \beta_1(\boldsymbol{x}_1)_{it} + \beta_2(\boldsymbol{x}_2)_{it} + v_i + \varepsilon_{it}, \tag{3}$$

with $i = 1, \ldots, m$ the cross-sectional index, and $t = 1, \ldots, T$ the time index. The $v_i$ are effects specific to cross-sectional unit $i$, and the $\varepsilon_{it}$ are mutually independent idiosycratic effects.

Obtain estimates of $\beta_1$ and $\beta_2$, first using the fixed-effects model, and then the random-effects model. For the fixed-effects model, find estimates of the $v_i$, $i = 1, \ldots, m$.

Give the estimates for the within-group estimator, and for the between-groups estimator, explaining your reasoning.

**4.** In the model (3) above, suppose that the $\varepsilon_{ti}$ are white noise with variance $\sigma_\varepsilon^2$ and that the $v_i$ are white noise with variance $\sigma_v^2$. Consider the regression that yields the between-groups estimator, equation (9.57) in the text, and show that the variance of the disturbances in this equation is $\sigma_v^2 + \sigma_\varepsilon^2/T$.

The random-effects estimator relies on feasible GLS, with a disturbance covariance estimator $\boldsymbol{\Omega}$ which is block-diagonal with blocks $\boldsymbol{\Sigma}$ given by $\boldsymbol{\Sigma} = \sigma_\varepsilon^2 \mathbf{I}_T + \sigma_v^2 \boldsymbol{\iota}\boldsymbol{\iota}^\top$; see equation (9.56). Show that

$$\boldsymbol{\Sigma}^{-1/2} = \frac{1}{\sigma_\varepsilon}(\mathbf{I}_T - \lambda \boldsymbol{P}_{\boldsymbol{\iota}}),$$

with

$$\lambda = 1 - \left(\frac{T\sigma_v^2}{\sigma_\varepsilon^2} + 1\right)^{-1/2}.$$

**5.** The file **https://russell-davidson.research.mcgill.ca/data/classical.data** contains simulated data, presented in four columns, of which the first is just the index of the observations, from 1 to 50. The following three columns contain, in order, observations on the dependent variable $\boldsymbol{y}$, and two exogenous explanatory variables $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. Details of the DGP are found at the end of the file.

Run the regression for the classical normal linear model $\boldsymbol{y} = \beta_0 + \beta_1\boldsymbol{x}_1 + \beta_2\boldsymbol{x}_2 + \boldsymbol{u}$ by OLS, and construct an equal-tailed 95% confidence interval for the variance parameter $\sigma^2$.

Since we are assuming the restrictions of the classical normal linear model, tests and confidence intervals are exact. If the disturbances were not normally distributed, how could one obtain a bootstrap confidence interval?

**6.** There are 100 observations on three variables in the file **https://russell-davidson.research.mcgill.ca/data/ar1.data** The first is just the time index, running from 1 to 100; the next two are denoted $\boldsymbol{x}$ and $\boldsymbol{y}$. The model to be estimated is as follows:

$$y_t = a + bx_t + u_t, \qquad u_t = \rho u_{t-1} + \varepsilon_t. \qquad \boldsymbol{\varepsilon} \sim \mathrm{IID}(0, \sigma^2). \tag{4}$$

A first step is to test the hypothesis that $\rho = 0$, meaning that there is no serial correlation of the disturbances. Compute a $P$ value for a suitable test.

Suppose now that the null hypothesis that $\rho = 0$ is rejected. One way to estimate the model parameters, $a$, $b$, $\rho$, and $\sigma^2$ is to rewrite (4) as

$$y_t = a(1-\rho) + bx_t + \rho y_{t-1} - b\rho x_{t-1} + \varepsilon_t,$$

and then to estimate this transformed model by nonlinear least squares with $t = 2, \ldots, 100$. What parameter estimates do you get in this way?

The other way to proceed is to start with a consistent estimate of $\rho$ and estimate the parameters by feasible generalised least squares (FGLS). Find parameter estimates by FGLS with $t = 2, \ldots, 100$. However, the information in the first observation can also be used when appropriately transformed. Obtain parameter estimates that incorporate the first observation.

**7.** Repeat the test of the null hypothesis according to which $\rho = 0$ in the model (4), by use of the bootstrap. Do this in two ways: with a parametric bootstrap, and a resampling bootstrap. Write down the bootstrap DGPs explicitly for these two methods. Bear in mind the two Golden Rules of bootstrapping! Is there any conflict with the asymptotic test in the previous question and these two bootstrap tests?