Errata for
Stochastic Calculus for Finance II:
Continuous-Time Models
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These are corrections to the 2008 printing.

Page XIX, line 2. Insert the word "and" between "finance" and "is essential."

Page XIX, line 5. Change *Early Exercise* to *American Derivative Securities.*

Page 15, lines 1-2. Change the text to
the maximal distance between two successive y_k partition points,

Page 44, line 8. The limit should be as $n \to \infty$, so that the expression is

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{e^{tX} - e^{s_n X}}{t - s_n}\right],$$

Page 44, lines 5 and 4 from bottom. E should be $\mathbb{E}$ in the two equations $\mathbb{E}\big[|X|e^{tX}\big] < \infty$ and $\varphi'(t) = \mathbb{E}\big[Xe^{tX}\big]$.

Page 51, line 2. Change "there is a σ-algebra $\mathcal{F}(t)$" to "there is a σ-algebra $\mathcal{F}(t)$ of subsets of Ω."

Page 66, equation (2.3.3). Change E_N to $\mathbb{E}_N$ in the equation

$$\mathbb{E}_N[X](\omega_1 \dots \omega_N) = X(\omega_1 \dots \omega_N). \tag{2.3.3}$$

Page 67, equations (2.3.8)–(2.3.11). Change E_2 to $\mathbb{E}_2$ on the left-hand side of each of these four equations.

Page 68, equations (2.3.12)–(2.3.15). Change E_2 to $\mathbb{E}_2$ under the integral sign on the left-hand side of each of these four equations.

Page 68, line 11. Change E_2 to $\mathbb{E}_2$ under the integral sign on the left-hand side of this equation, so that the equation becomes

$$\int_{A_H} \mathbb{E}_2[S_3](\omega)\, d\mathbb{P}(\omega) = \int_{A_H} S_3(\omega)\, d\mathbb{P}(\omega).$$

Page 70, equation (2.3.22). Change E to $\mathbb{E}$ on the right-hand side of this equation, so that it becomes

$$\mathbb{E}\big[\varphi(X)\big|\mathcal{G}\big] \geq \varphi\big(\mathbb{E}[X|\mathcal{G}]\big). \tag{2.3.22}$$

Page 70, lines 7 and 6 from bottom. Change E to $\mathbb{E}$ in the expressions $\mathbb{E}[X|\mathcal{G}]$ and $\mathbb{E}[Y|\mathcal{G}]$.

Page 71, line 10 from bottom. There is a d missing before $\mathbb{P}(\omega)$ in the integral on the right-hand side of the equation. The equation should be

$$\int_A \mathbb{E}\big[\mathbb{E}[X|\mathcal{G}]\big|\mathcal{H}\big](\omega)\, d\mathbb{P}(\omega) = \int_A \mathbb{E}[X|\mathcal{G}](\omega)\, d\mathbb{P}(\omega) \text{ for all } A \in \mathcal{H}.$$

Page 75, line 10 from bottom. The second factor $\mathbb{P}\{X \in C\}$ on the right-hand side of the equation should be $\mathbb{P}\{Y \in C\}$.
Page 80, line 3. Change E to $\mathbb{E}$ in the equation $Y_2 = Y - \mathbb{E}[Y|X]$.
Page 105, line 4. The text should read
we sum both sides of (3.4.8)
Page 120, line 9. The right-hand side of the equation should be $e^{-2m|\mu|}$ so that the equation becomes $\mathbb{P}\{\tau_m < \infty\} = e^{-2m|\mu|}$.
Page 120, line 5 from bottom. Replace $X_{1,n}, \dots, X_{n,n}$ with $X_{1,n}, \dots, X_{nt,n}$.
Page 120, line 3 from bottom. Replace $k = 1, \dots, n$ with $k = 1, \dots, nt$.
Page 121, line 2. The term $M_{nT,n}$ should be $M_{nt,n}$, so that the line becomes

$$= S(0) \exp\left\{ \frac{\sigma}{2\sqrt{n}}(nt + M_{nt,n}) \right\} \exp\left\{ -\frac{\sigma}{2\sqrt{n}}(nt - M_{nt,n}) \right\}$$

Page 122, line 4 from bottom. The term $-ax^2$ on the right-hand side of the equation should be $-a^2x^2$, so that the equation becomes

$$I(a,b) = \frac{1}{2a} \int_0^\infty \left(a + \frac{b}{x^2} \right) \exp\left\{ -a^2x^2 - \frac{b^2}{x^2} \right\} dx.$$

Page 129. To be consistent with the notation elsewhere in the text, the curly braces $\{\cdots\}$ used for conditional expectations on this page should be brackets $[\cdots]$. Hence, the first displayed equation should be

$$\begin{aligned} &\mathbb{E}\Big[\Delta(t_j)\big(W(t_{j+1}) - W(t_j)\big)\Big|\mathcal{F}(s)\Big] \\ &= \mathbb{E}\Big[\mathbb{E}\big[\Delta(t_j)\big(W(t_{j+1}) - W(t_j)\big)\big|\mathcal{F}(t_j)\big]\Big|\mathcal{F}(s)\Big] \\ &= \mathbb{E}\Big[\Delta(t_j)\big(\mathbb{E}\big[W(t_{j+1})\big|\mathcal{F}(t_j)\big] - W(t_j)\big)\Big|\mathcal{F}(s)\Big] \\ &= \mathbb{E}\Big[\Delta(t_j)\big(W(t_j) - W(t_j)\big)\Big|\mathcal{F}(s)\Big] \;=\; 0, \end{aligned}$$

the second displayed equation should be

$$\mathbb{E}\left[\sum_{j=\ell+1}^{k-1} \Delta(t_j)\big[W(t_{j+1}) - W(t_j)\big] \middle| \mathcal{F}(s) \right] = 0,$$

and the third displayed equation should be

$$\begin{aligned} &\mathbb{E}\Big[\Delta(t_k)\big(W(t) - W(t_k)\big)\Big|\mathcal{F}(s)\Big] \\ &= \mathbb{E}\Big[\mathbb{E}\big[\Delta(t_k)\big(W(t) - W(t_k)\big)\big|\mathcal{F}(t_k)\big]\Big|\mathcal{F}(s)\Big] \\ &= \mathbb{E}\Big[\Delta(t_k)\big(\mathbb{E}\big[W(t)\big|\mathcal{F}(t_k)\big] - W(t_k)\big)\Big|\mathcal{F}(s)\Big] \\ &= \mathbb{E}\Big[\Delta(t_k)\big(W(t_k) - W(t_k)\big)\Big|\mathcal{F}(s)\Big] \;=\; 0. \end{aligned}$$

Page 130, line 13. The second sum in the first line of equation (4.2.7) should have lower limit of summation $j = 0$, so that the line becomes

$$\mathbb{E}I^2(t) = \sum_{j=0}^{k} \mathbb{E}\big[\Delta^2(t_j)D_j^2\big] = \sum_{j=0}^{k} \mathbb{E}\Delta^2(t_j) \cdot \mathbb{E}D_j^2$$

Page 139, lines 2 and 1 from bottom. Change the text to the following:

If we take a function $f(t,x)$ of both t and x and assume that all the first- and second-order derivatives of $f(t,x)$ exist, then Taylor's Theorem says that

Page 141, line 10. Insert the sentence:

Because the terms involving the partial derivatives f_{tx} and f_{tt} contribute zero to the final answer, it turns out not to be necessary to assume that these derivatives exist.

The sentence in the text, "The higher-order terms likewise contribute zero to the final answer," then follows.

Page 143, line 6. The upper limit of integration on the last integral in this line should be T, not t, so that the line becomes

$$= \int_0^T f'\big(W(t)\big)\, dW(t) + \frac{1}{2}\int_0^T f''\big(W(t)\big)\, dt$$

Page 145, first line of the footnote. Change E to $\mathbb{E}$ in the expression $\mathbb{E}\int_0^t \Gamma^2(u)\Delta^2(u)\, du$.

Page 180, line 12. There is a right-parenthesis missing in the expression

$$-\frac{b(t_j - t_{j-1})}{\tau_j\tau_{j-1}}.$$

The line should be

$$= \sum_{j=1}^{n} \frac{\tau_j\tau_{j-1}}{t_j - t_{j-1}} \left(\frac{x_j}{\tau_j} - \frac{x_{j-1}}{\tau_{j-1}} - \frac{b(t_j - t_{j-1})}{\tau_j\tau_{j-1}}\right)^2$$

Page 198, line 2. The line should be

processes $W_1(t)$ and $W_2(t)$ such that $W_2(0) = 0$,

Page 203, equation (4.10.37). $Y_1(t_0)$ should be $Y_i(t_0)$, so that the equation becomes

$$\lim_{\epsilon\downarrow 0} \mathbb{E}\big[|Y_i(t_0+\epsilon) - Y_i(t_0)|\big|\mathcal{F}(t_0)\big] = 0. \qquad (4.10.37)$$

Page 203, line 9 from bottom. The line should end with a right parenthesis to close the parentheses opened in the second line from the bottom of page 202, so that the line becomes

You may use (4.10.37) without proving it.)

Page 206, line 6 from bottom. The term $|f''(x)|$ should be $|f_n''(x)|$.

Page 236, line 4 from bottom. There is a du missing in the factor $e^{\int_0^t A(u)du}$. The left-hand side of the equations should be

$$e^{\int_0^t A(u)du} D(t)S(t)$$

Page 238, line 12. Replace $0 < t_1 < t_2 < t_n < T$ with

$$0 < t_1 < t_2 < \cdots < t_n < T$$

Page 257, line 5 from bottom. There is a dt missing after $\rho_{ik}(t)$. The equation in this line should be $dB_i(t)\,dB_k(t) = \rho_{ik}(t)\,dt$.

Page 258, line 1. There is a dt missing at the end of this equation. The equation should be

$$d\widetilde{B}_i(t)\,d\widetilde{B}_k(t) = \rho_{ik}(t)\,dt.$$

Page 267, line 16. $E^{t,x}$ should be $\mathbb{E}^{t,x}$.

Page 279, lines 6–7. Replace this line with the text:

For the process $Y(u)$, we have the equation

$$dY(u) = S(u)\,du. \tag{6.6.6}$$

Page 279, line 11-12. Replace these lines with the text:

Note that $Y(u)$ alone is not a Markov process because its equation (6.6.6) involves the process $S(u)$. However, the pair $(S(u), Y(u))$

Page 293, line 8. The dt in the first term on the right-hand side of the equation should be du, so that the equation becomes

$$dS(u) = rS(u)\,du + \sigma\big(u, S(u)\big)S(u)\,d\widetilde{W}(u),$$

Page 296, line 4 from bottom. The random variable $Z(T)$ under the integral should be $\widehat{Z}(T)$, so that the equation becomes

$$\widehat{\mathbb{P}}(A) = \int_A \widehat{Z}(T)\,d\widetilde{\mathbb{P}} \text{ for all } A \in \mathcal{F}.$$

Page 296, line 3 from bottom. Insert a space between Theorem and 5.2.3.

Page 301, line 15 from bottom. "Exercise 7.8" should be "Exercise 7.1."

Page 313, line 1. The factor $\frac{\partial}{\partial y}(\frac{x}{y})$ in the middle of this equation should be $\frac{\partial}{\partial x}(\frac{x}{y})$, so that the equation becomes

$$v_{xx}(t,x,y) = u_{zz}\left(t, \frac{x}{y}\right) \cdot \frac{\partial}{\partial x}\left(\frac{x}{y}\right) = \frac{1}{y} u_{zz}\left(t, \frac{x}{y}\right),$$

Page 314, line 3 from bottom. There is a minus sign missing in the equation in this line. The equation should be $-e^{rt}e^{-rt}S(t) = -S(t)$.

Page 334, equation (7.8.17). The expectation operator $\mathbb{E}$ should be $\widetilde{\mathbb{E}}$, so that the equation becomes

$$\widetilde{\mathbb{E}}\big[f\big(S(T),Y(T)\big)\big|\mathcal{F}(t)\big] = g\big(S(t),Y(t)\big). \tag{7.8.17}$$

Page 356, line 11. Replace τ^* with τ_{L_*} in four places, so that the equation becomes

$$v\big(S(0)\big) = \widetilde{\mathbb{E}}\Big[e^{-r\tau_{L_*}}v\big(S(\tau_{L_*})\big)\Big] = \widetilde{\mathbb{E}}\Big[e^{-r\tau_{L_*}}\big(K - S(\tau_{L_*})\big)\Big].$$

Page 357, line 8 from bottom (counting footnote). Replace "in the next subsection" with "below", so that the line becomes

the put price provided below. It is known that $L(T)$ decreases

Page 361, line 2. Replace τ^* with τ_* in the factor $e^{-r(t\wedge\tau_*)}$ appearing in the term $e^{-r(t\wedge\tau_*)}v(t\wedge\tau_*, S(t\wedge\tau_*))$.

Page 363, line 18. Insert the word "nondecreasing" after "convex," so that the line becomes

convex nondecreasing function of a submartingale and, because of Jensen's inequality, this

Page 366, line 16. Replace $h_n(S(t_n)-)$ with $h_n(S(t_n-))$.

Page 396, line 8 from bottom. The open bracket [should be before rather than after log, so the line becomes

$$= \widetilde{\mathbb{P}}^T\left\{\frac{\widetilde{W}^T(T)}{\sqrt{T}} > \frac{1}{\sigma\sqrt{T}}\left[\log\frac{KB(0,T)}{S(0)} + \frac{1}{2}\sigma^2 T\right]\right\}$$

Page 406, line 1. Remove the hyphen in yield-models.

Page 409, line 8 from bottom. Change Theorem 4.5.4 to Theorem 4.6.5.

Page 448, equation (10.2.5). Replace $d\widetilde{W}_1(t)$ with $d\widetilde{W}_2(t)$, so the equation becomes

$$dY_2(t) = -\lambda_{21}Y_1(t)\,dt - \lambda_2 Y_2(t)\,dt + d\widetilde{W}_2(t), \tag{10.2.5}$$

Page 452, lines 5-9 from bottom. In each of these five equations, $\mathbb{E}$ on the left-hand side should be $\widetilde{\mathbb{E}}$, so that the equations become

$$\begin{aligned}
\widetilde{\mathbb{E}}I_2^2(t) &= \frac{1}{2\lambda_2}\big(e^{2\lambda_2 t} - 1\big),\\
\widetilde{\mathbb{E}}\big[I_2(t)I_3(t)\big] &= 0,\\
\widetilde{\mathbb{E}}\big[I_2(t)I_4(t)\big] &= \frac{t}{\lambda_1+\lambda_2}e^{(\lambda_1+\lambda_2)t} + \frac{1}{(\lambda_1+\lambda_2)^2}\big(1 - e^{(\lambda_1+\lambda_2)t}\big),\\
\widetilde{\mathbb{E}}I_3^2(t) &= \frac{1}{\lambda_2}\big(e^{2\lambda_2 t} - 1\big),\\
\widetilde{\mathbb{E}}\big[I_3(t)I_4(t)\big] &= 0.
\end{aligned}$$

Page 455, equation (10.7.14). There is a dt missing after $-\Lambda Y(t)$ on the right-hand side of the equation. The equation should be

$$dY(t) = -\Lambda Y(t)\,dt + d\widetilde{W}(t), \tag{10.7.14}$$

Page 456, equation (10.7.16). There should be a σ multiplying $d\widetilde{W}_1(t)$ on the right-hand side of the equation. The equation should be

$$dR(t) = \big(a - bR(t)\big)\,dt + \sigma\,d\widetilde{W}_1(t) \tag{10.7.16}$$

Page 456, line 8 from bottom. The line should be

Define a, b, and σ in terms of the parameters in (10.2.4)–(10.2.6).

Page 518, line 8. There is a τ missing in the term $e^{-\tilde{\beta}\tilde{\lambda}\tau}$. The line should be

$$= \widetilde{\mathbb{E}}\Bigg[\widetilde{\mathbb{E}}\Bigg[e^{-r\tau}\Bigg(xe^{-\tilde{\beta}\tilde{\lambda}\tau}\exp\left\{-\sigma\sqrt{\tau}\,Y + \left(r - \frac{1}{2}\sigma^2\right)\tau\right\}$$

Page 518, line 3 from bottom. There is a τ missing in the term $e^{-\tilde{\beta}\tilde{\lambda}\tau}$. The line should be

$$\widetilde{\mathbb{E}}\Bigg[e^{-r\tau}\Bigg(xe^{-\tilde{\beta}\tilde{\lambda}\tau}\exp\left\{-\sigma\sqrt{\tau}\,Y + \left(r - \frac{1}{2}\sigma^2\right)\tau\right\}$$

Page 519, equation (11.7.32). Replace $\mathbb{E}$ on the right-hand side of the equation with $\widetilde{\mathbb{E}}$, so that the equation becomes

$$c(t,x) = \widetilde{\mathbb{E}}\,\kappa\left(\tau, xe^{-\tilde{\beta}\tilde{\lambda}\tau}\prod_{i=N(t)+1}^{N(T)}(Y_i+1)\right). \tag{11.7.32}$$

Page 519, line 6. Replace $\mathbb{P}$ with $\widetilde{\mathbb{P}}$ on the left-hand side of the equation, so that the equation becomes

$$\widetilde{\mathbb{P}}\big\{N(T) - N(t) = j\big\} = e^{-\widetilde{\lambda}\tau}\frac{\widetilde{\lambda}^j\tau^j}{j!}. \qquad \square$$

Page 526, line 7. Insert the word "independent" before "Poisson," so that the line becomes

Exercise 11.4. Suppose $N_1(t)$ and $N_2(t)$ are independent Poisson processes with inten-

Page 532, line 2. The equation should be

$$\frac{7}{9} = \frac{2}{3} + \frac{0}{9} + \frac{2}{27} + \frac{2}{81} + \frac{2}{243} + \cdots.$$

Page 539, citation 61. The citation should be

61. Dupire, B. (1994) Pricing with a smile, *Risk* **7** (1), 18-20.